HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Optimal forest harvesting under stochastic rate of interest

Saeed Bayazidi

Olli Tahvonen

Department of Forest Science University of Helsinki

Introduction

Previous studies on stochastic rotation problem

- Stochastic forest growth (Hool 1966,…)
- Stochastic timber price (Norström 1975,…)
- Stochastic forest value (Brock and Rothschild 1984,…)

However, the rate of interest plays a vital role in the rotation problem and is clearly stochastic

In Finland the optimal rotation varies between 60 and 130 years when the rate of interest have been varied between 0 and 11% in last 2 decades

A forest owner: “How should I react if the rate of interest temporarily jumps up to 15%?”

Previous Studies

1- Study by Alvarez and Koskela (2003):

Method: Continuous time optimal stopping Single stand “Wicksellian” rotation model :Stochastic rate of interest, assume:

$$dr_t = \alpha r_t (1 - \gamma r_t) dt + \sigma r_t dw, \ r_0 \ given$$

$$dx_t = \mu x_t dt + \sigma x_t dw, \ x_0 \ given \quad (GBM \ for \ forestvalue)$$

Results:

- Stochastic interest rate lengthen optimal rotation
- Clear-cut when the rate of interest exceeds some level $r^* > \mu$

Note: Cut independently on forest age (!)

2- Study by Buongiorno and Zhou (2009)

Method- discrete state MDP as Stochastic Faustmann formula

Results:

-Expected maximum net present value under stochastic interest rate is higher than net present value obtained with fixed interest rate

- But effect of variation of interest rate on net present value is not large

The approach of this study:

-Faustmann framework with multiple stands
Mitra and Wan (1985), Salo and Tahvonen (2003)
-optimal consumption- savings decision making
cf. Merton optimal portfolio-consumption savings model (1971)
-discrete time
-stochastic rate of interest follows a discrete time discrete state process

$$r_{t+1} = \eta(\bar{r} - r_t) + \varepsilon_{t+1} r_t,$$

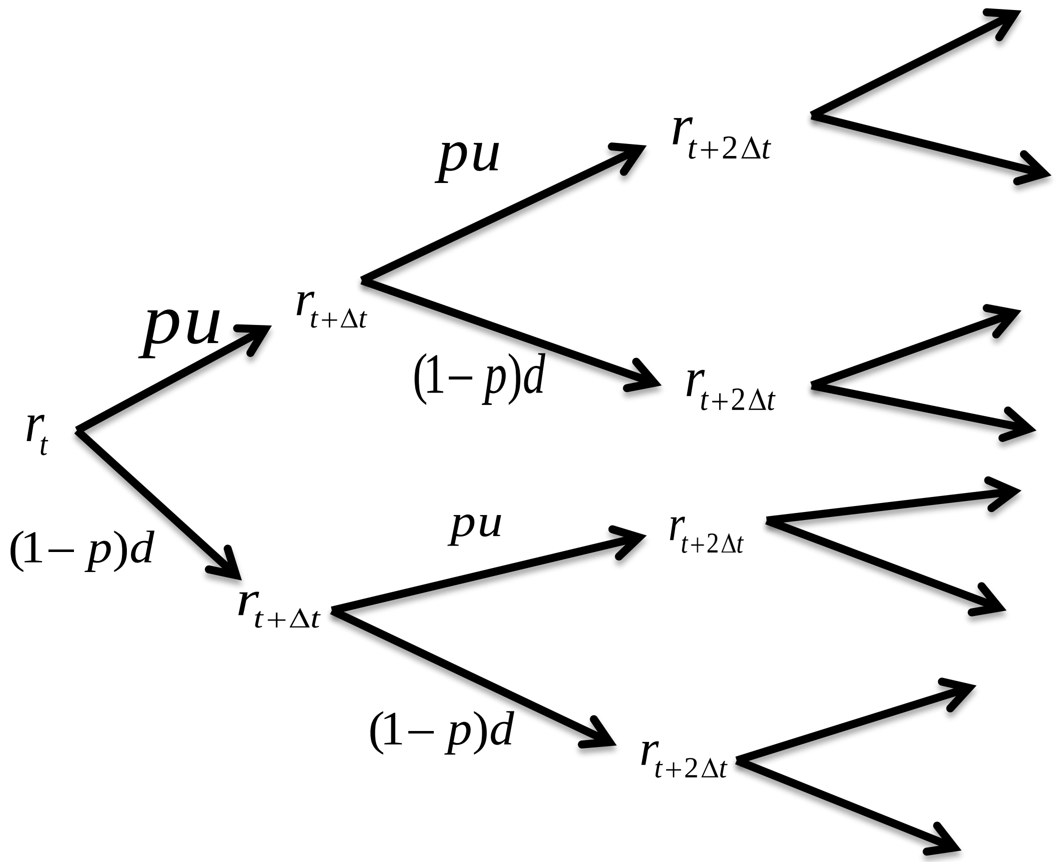
where ε_{t+1} *may take two discrete values u and d*

let q denote the probability of u and $1-q$ *the probability of d*

If $\eta = 0$, *discrete time discrete state random walk*

If $\eta > 0$, *one form of Ornstein–Uhlenbeck mean reverting process*

Simulation



$$t \longrightarrow r_{t+\Delta t} \longrightarrow r_{t+2\Delta t} \longrightarrow r_{t+3\Delta t} \;,,,\; \longrightarrow r_{t+n\Delta t}$$

The model: multiple stands, consumption- savings decision making

$$\max \quad J = E\left\{ \sum_{t=0}^{T} U(c_t^i)\ b^t \right\}$$

$$r_{t+1} = \eta(\bar{r} - r_t) + \varepsilon_{t+1} r_t, t = 0,\ldots,t+1,\ r_0 \ \ given,$$

$$a_{t+1}^i = a_t^i(r_t^i + 1) + h_t^i(p - v) - wy_t^i + m - c_t^i,\ \ a_0 \ \ given,\ t = 0,\ldots,T, i = 1,\ldots,W,$$

$$y_t^i = \sum_{s=1}^{n} z_{st}^i,\ h_t^i = \sum_{s=1}^{n} f_s z_{st}^i,\ \ f_{s+1} \geq f_s,\ s = 1,\ldots,n-1,\ t = 0,\ldots,T,\ i = 1,\ldots,W,$$

$$x_{s+1,t}^i = x_{st}^i - z_{st}^i, s = 1,\ldots,n-2,\ t = 0,\ldots,T,\ i = 1,\ldots,W,$$

$$x_{n,t+1}^i = x_{nt}^i - z_{nt}^i + x_{n-1,t}^i - z_{n-1,t}^i, t = 0,\ldots,T,\ i = 1,\ldots,W,$$

$$x_{1t+1}^i = A - \sum_{s=2}^{n} z_{s,t+1}^i, t = 0,\ldots,T, i = 1,\ldots,W,$$

$$x_{st}^i \geq 0,\ t = 0,\ldots,T+1,\ i = 1,\ldots,W,$$

$$z_{st}^i \geq 0,\ t = 0,\ldots,T,\ i = 1,\ldots,W,$$

$$a_{T+1}^i \geq 0,\ i = 1,\ldots,W,$$

$$x_{so},\ s = 1,\ldots,n \ \ given.$$

cf. optimal portfolio selection – consumption savings problem by Merton 1971

- Solution method: stochastic programming
  - -origin in stochastic LP
  - -extension to nonlinear problems: e.g. Rockafellar and Wets, 1975, Stochastic convex programming, Kuhn-Tucker conditions, J. of Math. Econom.

The stochastic variable r_t may take $2^T \equiv W$ different time paths
Denote the time paths as *i*=1,…,W and specify different state and control variables for all the time paths

Let μ_i denote the probability of price path i. Then the expected utility to be maximized equals

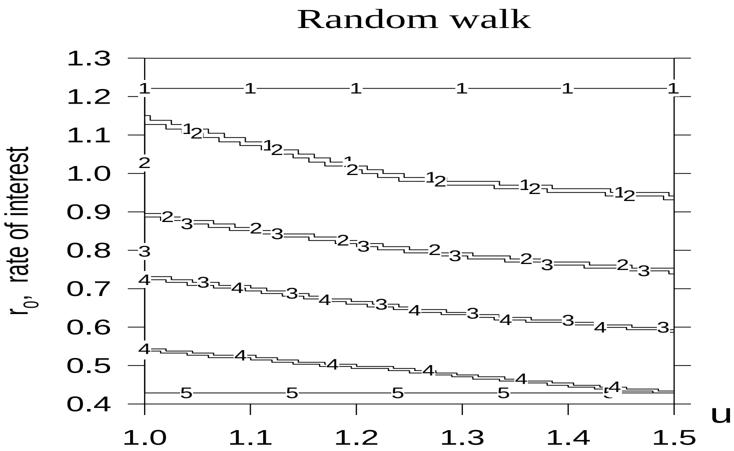
$$J = \sum_{i=1}^{W} \mu_i \sum_{t=0}^{T} U(c_t^i) b^t$$

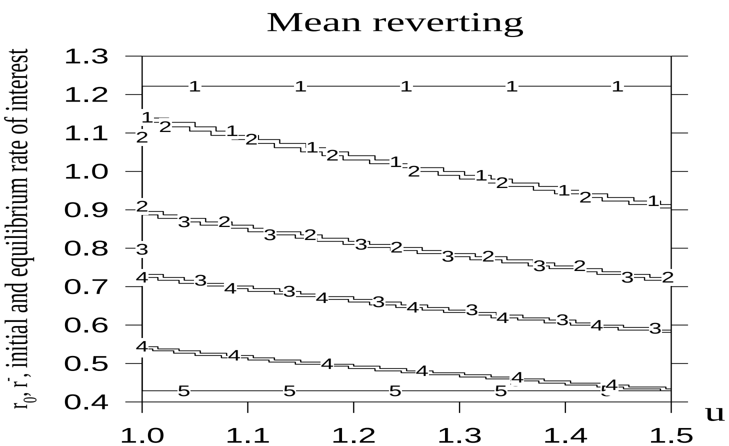
Apply

$$U = \frac{c^{1-\alpha}}{1-\alpha}, \; 0 \le \alpha \le 1$$

Results

Case of Risk Neutrality

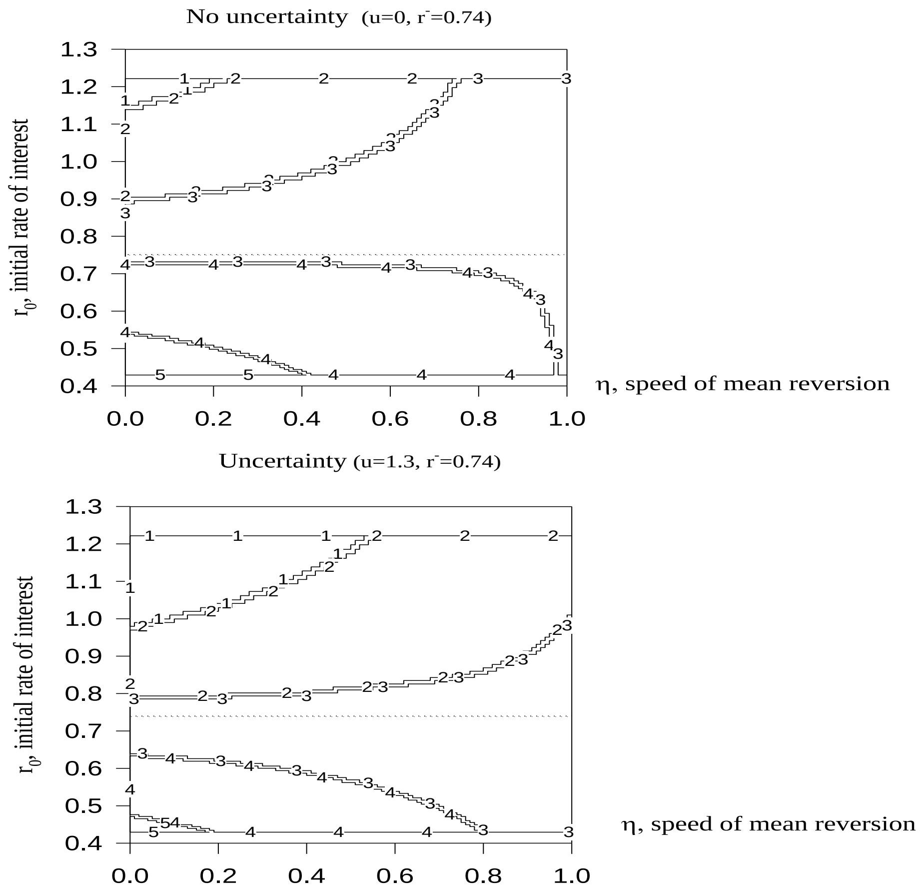




Note: c=0, w=0, p=40, f=9.12,28.65,64.18,122.31, 202.78, bs=0.25, a_0=1, T=10,

Both random walk and mean reverting interest rate shorten optimal rotation

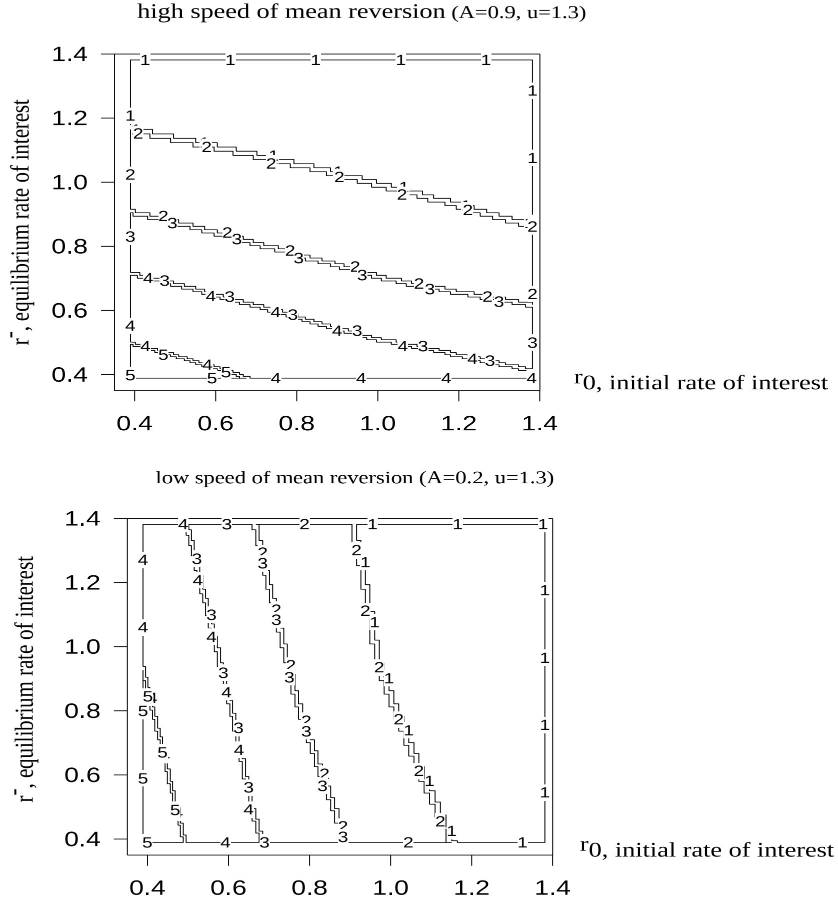
Case of Risk Neutrality cont,



Note: c=0, w=0, p=40, f=9.12,28.65,64.18,122.31, 202.78, bs=0.25, a_0=1, T=10,

under uncertainty and mean reversion the rotation is shorter than under certainty and mean reversion

Case of Risk Neutrality, cont,



Note: c=0, w=0, p=40, f=9.12,28.65,64.18,122.31, 202.78, bs=0.25, a_0=1, T=10,

Higher speed toward the equilibrium, fluctuations matter less

Why uncertainty of the interest rate Shortens rotation?

Assume forest owner is a borrower and that subjective discount rate exceeds the market rate of interest

Incentive to consume immediately as much as possible and pay back the loan later

However, the budget constraint must be met

In the case of high interest rate time path borrowing is limited since interest cost must be paid with future forest income

When a possible interest rate is high enough borrowing approaches zero and it becomes optimal to finance present consumption by earlier cuttings than under certainty

Why uncertainty of the interest rate Shortens rotation?

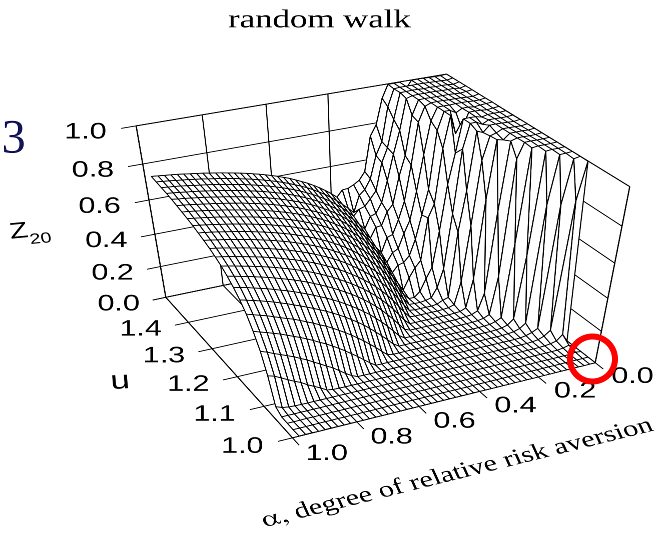
Assume the forest owner is a saver and that the subjective rate of discount is low compared to the market rate of interest

If under stochasticity the growth rate of the expected value of financial assets be less than the growth rate of financial assets under deterministic rate of interest

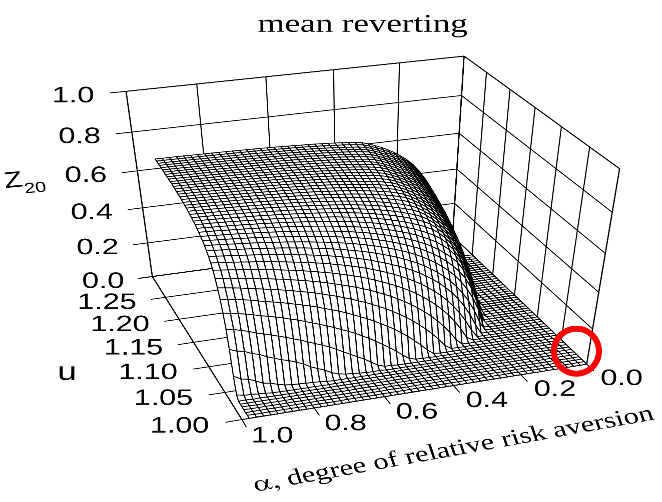
Thus to finance future consumption it may be optimal to harvest younger stands

Case of Risk Aversion

Initial state: x=[0,0,1,0,0], Faustmann rotation 3



Risk aversion implies smoothing of the age class structure, younger age classes may be harvested

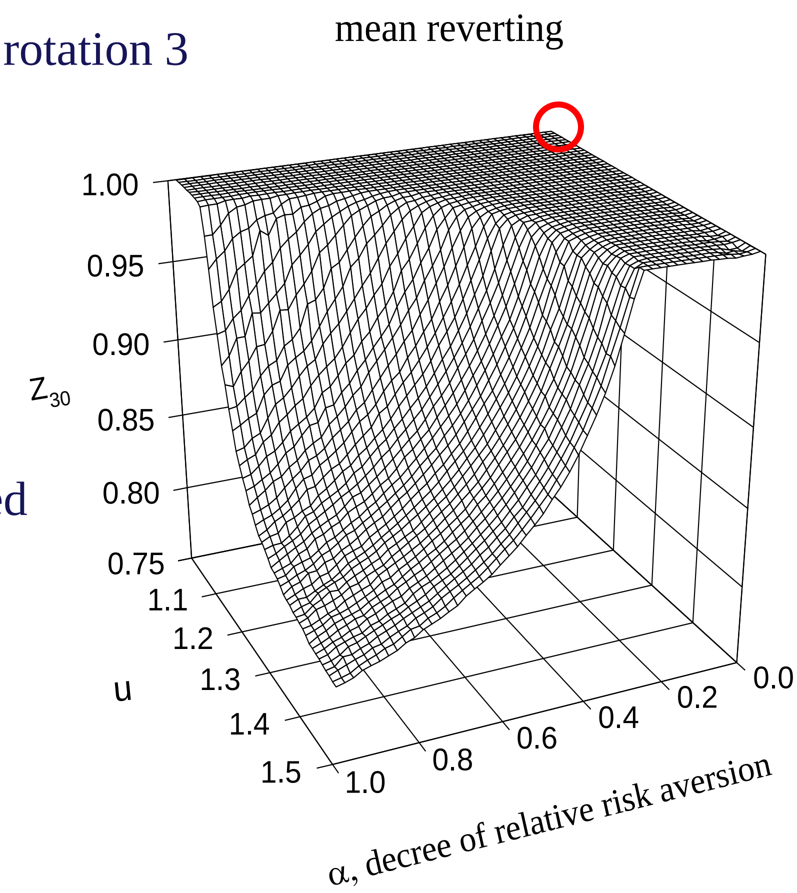


bs=.256, bm=bs ◯ - Faustamnn Solution

Case of Risk Aversion , cont,

Initial state: x=[0,0,1,0,0], under Faustmann rotation 3

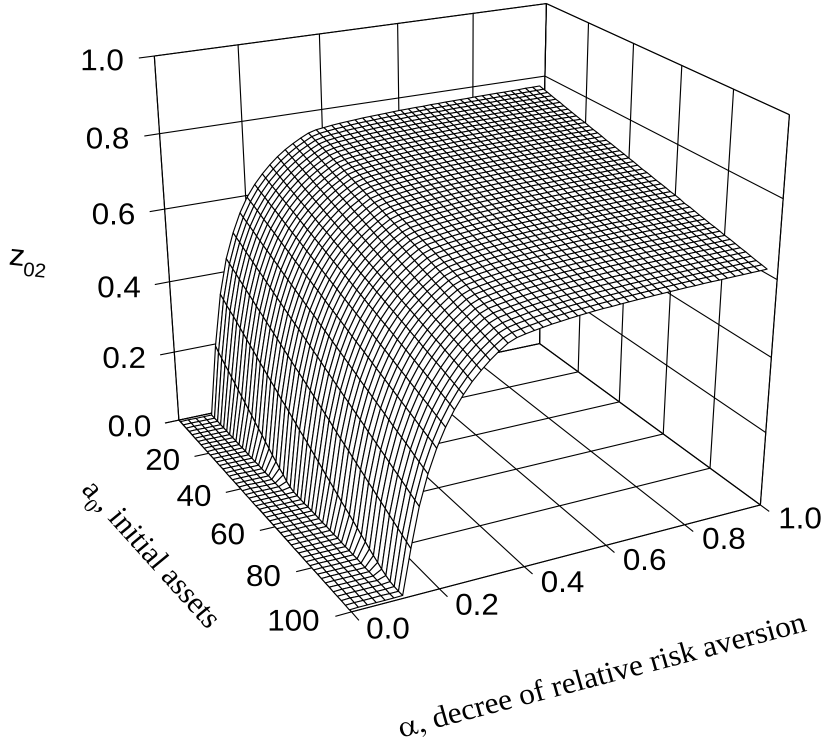
Smoothing may imply that trees are harvested at older ages than under certainty



Case of Risk Aversion , cont,

initial state x=[0,1,0,0,0], Faustmann rotation 2

mean reverting

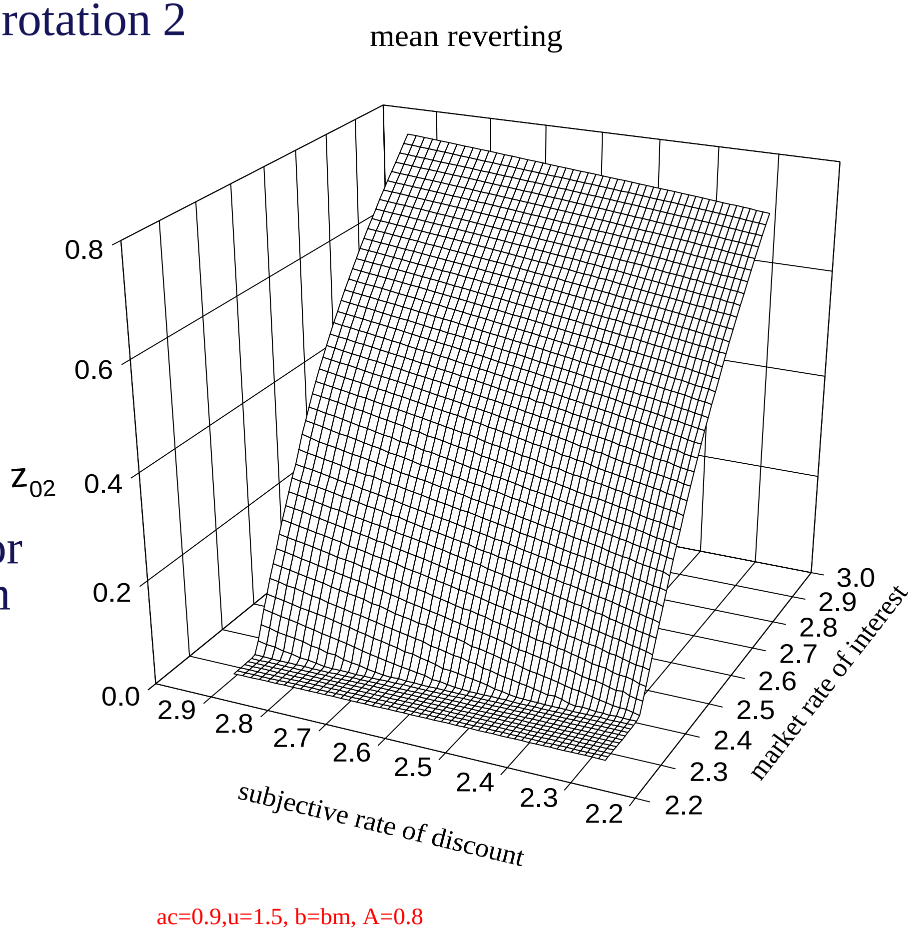


Higher initial assets decreases smoothing

Case of Risk Aversion , cont,

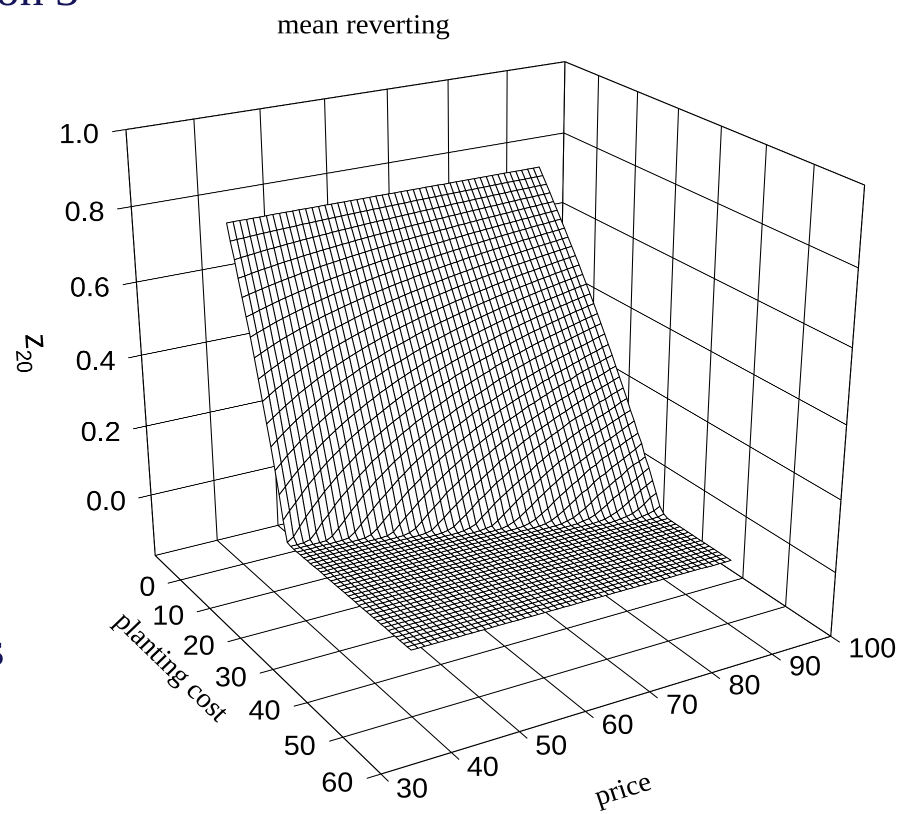
initial state x=[0,1,0,0,0], Faustmann rotation 2

Increasing subjective discount factor or the market rate of interest (equilibrium and initial) increases initial cutting



Case of Risk Aversion , cont,

initial state x=[0,0,1,0,0], Faustmann rotation 3



Higher price or lower planting cost increases initial cuttings

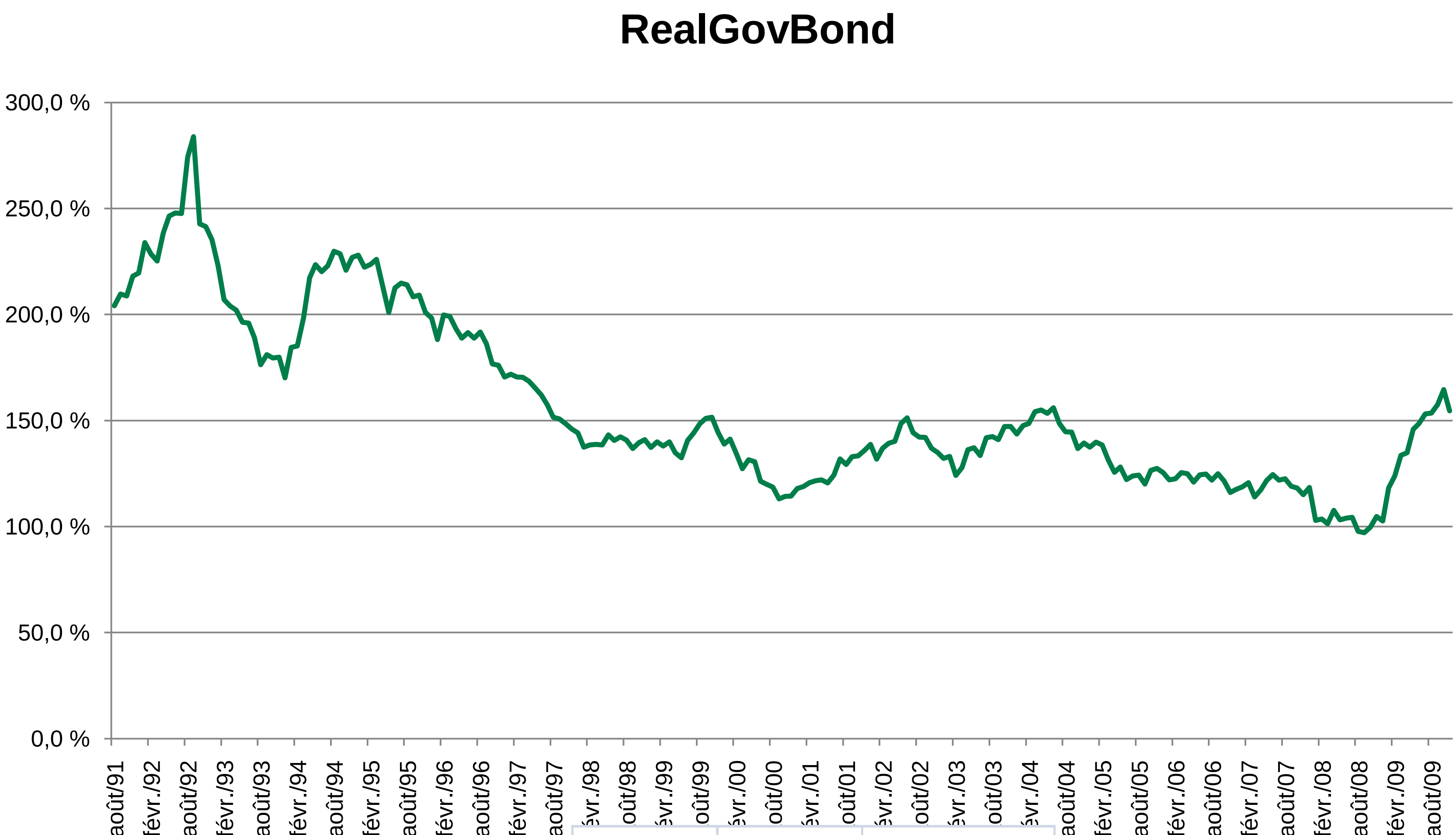
ac=0.8, bm=0.256=bs, u=1.3, a0=1, c=0, f=0.8,..,x20=1

Preliminary Conclusions

- Under risk neutrality stochastic interest rate implies shorter rotations
- Under risk aversion stochastic interest rate implies that it becomes optimal to smooth the age class structure
- Under risk aversion optimal rotation will be depend on the initial state of the forest and it may imply shorter rotation for younger forest stands or longer rotation for older forest stands (for robust conclusion Need to proceed to steady state?)
- Under risk aversion wealth, subjective time preference, etc. have effects on optimal rotation and timber supply

Merci

Projected Real interest over 10 years Finnish Government Bond yield



source: Bank of Finland

	10 year	Annual
average	156,1 %	4,6 %
Sd	39,8 %	2,5 %