Impact of the presence of risk of destructive event on forest silviculture

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Introduction

- Faustmann: what is the optimal duration of cycle production?
- Optimal production planning: best sequence of harvesting?
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Context

Actual context:

- Risk: frequency, amplitude
- Stand level
- Decision variables: thinning $h(.)$, cutting age $T$
- Tree averaged model
- Simple scenarii
Faustmann Rotation

Faustmann solution solves Rotation problems
A new rotation started at the same time as the previous ends
Focus on impact of rotation to harvesting
Faustmann solution based on dynamic models

$\max_{h(\cdot), T} J_0$
Population Dynamic Model

Tree-averaged Model:

Tree-number $n(.)$: \[
\frac{dn(t)}{dt} = -(m(t) + h(t))n(t)
\]

Tree-section at 1m30 $s(.)$: \[
\frac{ds(t)}{dt} = G(s(t), n(t), t)
\]

Demography: Natural mortality: $m$, Harvesting rate: $h(t) \leq \bar{h}$.

Economy: Price function: $p(s)$, Actualization: $\delta$.

Optimization problem:

\[
\max_{h(.), T} \mathcal{H}(h(.), T) + \mathcal{V}_0(T)
\]
(Thinning revenue + Final revenue)
Faustmann Rotation

Without risk:

Land value: \( J_0 = \sum_{i=1}^{\infty} \left[ V(h(\cdot), T) - c_1 \right] e^{-i\delta T} \)

\[ J_0 = (J_0 + V(h(\cdot), T) - c_1) e^{-\delta T} \]

where: \( V(h(\cdot), t) = \mathcal{H}(h(\cdot), t) + V_0(t) \)

Faustmann problem:

\[
\max_{h(\cdot), T} J_0 = \frac{\mathcal{H}(h(\cdot), T) + V_0(T) - c_1}{e^{\delta T} - 1}
\]
Without risk:

Faustmann problem:

\[
\max_T J_0 = \frac{\nu(T) - c_1}{e^{\delta T} - 1}
\]

where \( \nu(T) \) is:

\[
\nu(T) = \max_{h(\cdot)} [\mathcal{H}(h(\cdot), T) + \nu_0(T)]
\]
In presence of risk

- Destructive events occur in a Poisson process at rate $\lambda$ (average rate per unit time)
- Salvageable proportion $1 - \theta_t$
- Rate depreciation of timber due to an influx of wood on the market $\xi_t$.
- $\theta_t$ and $\xi_t$ correlated random variables (depending on the amplitude of the event)

\[
\alpha(t) = E(1 - \theta_t), \quad \alpha_p(t) = E((1 - \theta_t)(1 - \xi_t))
\]
In presence of a destructive event, three possibilities:

- At each event, cut and start a new rotation:
  \[0 \rightarrow M(0,t_1) \rightarrow t_1 \quad (t_1 \leq T)\]

- At each event, continue
  \[0 \rightarrow M(0,t_1) \rightarrow t_1 \rightarrow M(t_1,t_2) \rightarrow t_2 \rightarrow \ldots \rightarrow T\]

- Criteria:
  \[
  \begin{cases}
  \text{cut and start a new rotation} \\
  \text{continue}
  \end{cases}
  \]
In presence of risk : Case 1

\[ J_0 = \int_0^T (J_0 + V_1(h(\cdot), t) - c_1 - C_n(n(t), t))e^{-\delta t}dF(t) \]

(in case of event at time \( t \))

\[ + (J_0 + V(h(\cdot), T) - c_1)e^{-\delta T}(1 - F(T)) \]

(in case of no event before time \( T \))

where \( V_1(h(\cdot), t) = H(h(\cdot), t) + \alpha_p(t)V_0(t) \)

and \( C_n(n, t) = c_2 + c_n\theta_t n \)
In presence of risk : Case 1

\[
\max_T \frac{\delta + \lambda}{\delta} \frac{\tilde{V}(T) - c_1}{e^{(\delta + \lambda)T} - 1} - \frac{\lambda}{\delta} (c_1 + c_2)
\]

where : \( \tilde{V}(T) = \max_{h(\cdot)} \tilde{H}(h(\cdot), T) + V_0(T) \)

\[
\tilde{H}(h(\cdot), T) = \int_0^T \left[ p(s(t)) h(t) n(t) - \lambda c_n \alpha(t) n(t) + \lambda \alpha p(t) V_0(t) \right] e^{(\lambda + \delta)(T-t)} dt
\]

In comparison without risk :

\[
H(h(\cdot), T) = \int_0^T p(s(t)) h(t) n(t) e^{\delta(T-t)} dt
\]
No density dependent growth

Limit case

Using Pontryagin Maximum Principle:

- $m(t)$ is replaced by $m(t) + \lambda(1 - \alpha_p(t))$
- for a fixed rotation $T$, harvesting increases with risk in optimal management (increasing function of $\lambda(1 - \alpha(t))$

For respective optimal $T_*$ and with density dependence?
Eucalyptus stand (Saint-André 2002)

Growth: \( G(s, n, t) = \frac{0.7445(1 - e^{-0.482ns})}{n} dH(t) \)

where \( H \) is tree-high: \( dH(t) = e^{-\frac{t}{H_0}} \)

Weight (kg): \( w(s, h, t) = 0.29 + (127.8 + 0.32t)sh \)

Price function (Euro): \( p(s, t) = 0.01w(s, h(t), t) - 0.25 \)
**Eucalyptus stand**

\[ m = .0042, \lambda = .0075, \delta = .0034, \bar{h} = .0075 \text{ in month}^{-1} \]

<table>
<thead>
<tr>
<th>Risk</th>
<th>max</th>
<th>( h(.) )</th>
<th>( T )</th>
<th>( J_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0, \alpha_p = 0 ), 650 stems/ha</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>( h(.) ), ( T )</td>
<td>( h \equiv 0 )</td>
<td>58.5</td>
<td>3.29n_0</td>
</tr>
<tr>
<td>Yes</td>
<td>( T )</td>
<td>( h \equiv 0 \text{ fixed} )</td>
<td>54.0 (44.4)</td>
<td>1.04n_0</td>
</tr>
<tr>
<td>Yes</td>
<td>( h(.) ), ( T )</td>
<td>( h(t) = \bar{h}, t \geq 36.5 )</td>
<td>69.5 (54.2)</td>
<td>1.20n_0</td>
</tr>
<tr>
<td>( \alpha = .6, \alpha_p = .4 ), 650 stems/ha</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>( h(.) ), ( T )</td>
<td>( h \equiv 0 )</td>
<td>58.5</td>
<td>3.29n_0</td>
</tr>
<tr>
<td>Yes</td>
<td>( T )</td>
<td>( h \equiv 0 \text{ fixed} )</td>
<td>57.5 (46.7)</td>
<td>1.71n_0</td>
</tr>
<tr>
<td>Yes</td>
<td>( h(.) ), ( T )</td>
<td>( h(t) = \bar{h}, t \geq 43.5 )</td>
<td>65.5 (51.7)</td>
<td>1.77n_0</td>
</tr>
</tbody>
</table>

Higher \( T \), lower is the probability to finish the cycle

Self-insurance
Beech stand

**Growth**: \( G(s, n, t) = (1 - e^{-m_1 n\sqrt{4\pi s}})(m_2 + m_3 dH(t)) \)
where \( H \) is tree-high : \( dH(t) = m_4 H_{100} e^{-m_4 t} \) (Dhôte 1995)

**Volume Table** :
\[
v(d, h) = (a_0 d^2 h + a_1 dh + a_2 d^3 h^2)(1 + \frac{b_1}{d^3} + b_2 d^2 + \frac{b_3}{h} + b_4 h)
\]
(Bouchon 1982)

**Price function** (Euro) : for \( d > 7 \) (in cm)
\[
p(s, t) = (12 + 45(1 - (1 - \frac{d}{65})^{1.8})v(d, h(t)) \) (adjusted Tarp et al. 2000)
### Beech stand

LP = .01, δ = .02, \( \bar{h} = .2 \) in year\(^{-1} \)

<table>
<thead>
<tr>
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<th>( T )</th>
<th>( J_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.6, \alpha_p = 0.4, 4000 \text{ stems/ha}, H_{100} = 30 \text{ m} )</td>
<td>( h(t) = \bar{h}, t \in [0, 12.9] \cup [51.7, 57.3] )</td>
<td>81.9</td>
<td>1.23( n_0 )</td>
</tr>
<tr>
<td></td>
<td>No</td>
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<td></td>
<td>Yes</td>
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</tr>
<tr>
<td>( \alpha = 0.6, \alpha_p = 0.4, 4000 \text{ stems/ha}, H_{100} = 40 \text{ m} )</td>
<td>( h(t) = \bar{h}, t \in [0, 13.1] \cup [44.0, 49.4] )</td>
<td>75.6 (53.0)</td>
<td>0.67( n_0 )</td>
</tr>
</tbody>
</table>

### Respacing and thinning

<table>
<thead>
<tr>
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<th>( h(.) )</th>
<th>( T )</th>
<th>( J_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0.6, \alpha_p = 0.4, 4000 \text{ stems/ha}, H_{100} = 30 \text{ m} )</td>
<td>( h(t) = \bar{h}, t \in [0, 13.0] \cup [49.9, 55.5] )</td>
<td>75.3</td>
<td>2.21( n_0 )</td>
</tr>
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<td>Yes</td>
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</tr>
<tr>
<td>( \alpha = 0.6, \alpha_p = 0.4, 4000 \text{ stems/ha}, H_{100} = 40 \text{ m} )</td>
<td>( h(t) = \bar{h}, t \in [0, 13.0] \cup [42.4, 47.9] )</td>
<td>69.1 (49.9)</td>
<td>1.47( n_0 )</td>
</tr>
</tbody>
</table>
Conclusion

- Harvesting increases with risk
- Larger harvesting can induces larger final term
- Final term can be larger with risk
- Self-insurance -> we make endogenous the risk through optimization
- Species dependency
- Influence of model complexity
THANK YOU FOR YOUR ATTENTION

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