Impact of the presence of risk of destructive event on forest silviculture

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Introduction

- Faustmann : what is the optimal duration of cycle production ?
- Optimal production planning : best sequence of harvesting ?

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- Impact of risk of destructive events : storm, fire, ...
- Martell, Routledge (1980), Reed (1984 ...), Thorsen and Helles (1998), Stenger-Peyron (2005).

Context

Actual context :

- Risk : frequency, amplitude
- Stand level
- Decision variables : thinning $h(.)$, cutting age T
- Tree averaged model
- Simple scenarii

Faustmann Rotation

$$\begin{array}{ccccc} 0 & \longrightarrow & M(0,T) & \longrightarrow & T \\ \uparrow & & & & \downarrow \\ & \longleftarrow\longleftarrow & \longleftarrow & \longleftarrow\longleftarrow & \end{array}$$

- Faustmann solution solves Rotation problems
- A new rotation started at the same time as the previous ends
- Focus on impact of rotation to harvesting
- Faustmann solution based on dynamic models

$$\max_{h(.),T} J_0$$

Population Dynamic Model

Tree-averaged Model :

Tree-number $n(.)$: $$\frac{dn(t)}{dt} = -(m(t) + h(t))n(t)$$

Tree-section at 1m30 $s(.)$: $$\frac{ds(t)}{dt} = G(s(t), n(t), t)$$

Demography : Natural mortality : m, Harvesting rate : $h(t) \leq \overline{h}$.

Economy : Price function : $p(s)$, Actualization : δ.

Optimization problem :

$$\max_{h(.),T} \quad \mathcal{H}(h(.), T) \quad + \quad \mathcal{V}_0(T)$$

(Thinning revenue + Final revenue)

Faustmann Rotation

Without risk :

Land value : $J_0 = \sum_{i=1}^{\infty} [\mathcal{V}(h(.), T) - c_1] e^{-i\delta T}$

$$J_0 = (J_0 + \mathcal{V}(h(.), T) - c_1) e^{-\delta T}$$

where : $\mathcal{V}(h(.), t) = \mathcal{H}(h(.), t) + \mathcal{V}_0(t)$

Faustmann problem :

$$\max_{h(.),T} J_0 = \frac{\mathcal{H}(h(.), T) + \mathcal{V}_0(T) - c_1}{e^{\delta T} - 1}$$

Without risk :

Faustmann problem :

$$\max_{T} J_0 = \frac{\mathcal{V}(T) - c_1}{e^{\delta T} - 1}$$

where $\mathcal{V}(T)$ is :

$$\mathcal{V}(T) = \max_{h(.)} [\mathcal{H}(h(.), T) + \mathcal{V}_0(T)]$$

In presence of risk

- Destructive events occur in a Poisson process at rate λ (average rate per unit time)
- Salvageable proportion $1 - \theta_t$
- Rate depreciation of timber due to an influx of wood on the market ξ_t.
- θ_t and ξ_t correlated random variables (depending on the amplitude of the event)

$$\alpha(t) = E(1 - \theta_t), \qquad \alpha_p(t) = E((1 - \theta_t)(1 - \xi_t))$$

In presence of a destructive event, three possibilities :

- At each event, cut and start a new rotation :
  0 –> M(0,t_1) –> t_1 ($t_1 \leq T$)

- At each event, continue
  0 –> M(0,t_1) –> t_1 –> M(t_1,t_2) –> t_2 –> .. –> T

- Criteria : $\begin{cases} \text{cut and start a new rotation} \\ \text{continue} \end{cases}$

In presence of risk : Case 1

$$J_0 = \int_0^T (J_0 + \mathcal{V}_1(h(.), t) - c_1 - \mathcal{C}_n(n(t), t))e^{-\delta t} dF(t)$$

(in case of event at time t)

$$+(J_0 + \mathcal{V}(h(.), T) - c_1)e^{-\delta T}(1 - F(T))$$

(in case of no event before time T)

where $\mathcal{V}_1(h(.), t) = \mathcal{H}(h(.), t) + \alpha_p(t)\mathcal{V}_0(t)$
and $\mathcal{C}_n(n, t) = c_2 + c_n\theta_t n$

In presence of risk : Case 1

$$\max_T \frac{\delta+\lambda}{\delta} \frac{\widetilde{\mathcal{V}}(T) - c_1}{e^{(\delta+\lambda)T} - 1} - \frac{\lambda}{\delta}(c_1 + c_2)$$

where : $\widetilde{\mathcal{V}}(T) = \max_{h(.)} \widetilde{\mathcal{H}}(h(.), T) + \mathcal{V}_0(T)$

$\widetilde{\mathcal{H}}(h(.), T) =$

$$\int_0^T [p(s(t))h(t)n(t) - \lambda c_n \alpha(t) n(t) + \lambda \alpha_p(t) \mathcal{V}_0(t)] e^{(\lambda+\delta)(T-t)} dt$$

In comparaison without risk :

$$\mathcal{H}(h(.), T) = \int_0^T p(s(t))h(t)n(t)e^{\delta(T-t)} dt$$

No density dependent growth

Limit case

Using Pontryagin Maximum Principle :

- $m(t)$ is remplaced by $m(t) + \lambda(1 - \alpha_p(t))$
- for a fixed rotation T, harvesting increases with risk in optimal management (increasing function of $\lambda(1 - \alpha(t))$

For respective optimal T_* and with density dependence ?

Eucalytus stand

Eucalyptus stand (Saint-André 2002)

Growth : $G(s,n,t) = \frac{0.7445(1 - e^{-0.482ns})}{n} dH(t)$

where H is tree-high : $dH(t) = e^{-\frac{t}{H_0}}$

Weight (kg) : $w(s,h,t) = 0.29 + (127.8 + 0.32t)sh$

Price function (Euro) : $p(s,t) = .01w(s,h(t),t) - .25$

Eucalytus stand

$m = .0042$, $\lambda = .0075$, $\delta = .0034, \overline{h} = .0075$ in month^{-1}

Risk	max	$h(.)$	T	J_0
$\alpha = 0, \alpha_p = 0$, 650 stems/ha				
No	$h(.), T$	$h \equiv 0$	58.5	$3.29 n_0$
Yes	T	$h \equiv 0$ fixed	54.0 (44.4)	$1.04 n_0$
Yes	$h(.), T$	$h(t) = \overline{h}, t \geq 36.5$	69.5 (54.2)	$1.20 n_0$
$\alpha = .6, \alpha_p = .4$, 650 stems/ha				
No	$h(.), T$	$h \equiv 0$	58.5	$3.29 n_0$
Yes	T	$h \equiv 0$ fixed	57.5 (46.7)	$1.71 n_0$
Yes	$h(.), T$	$h(t) = \overline{h}, t \geq 43.5$	65.5 (51.7)	$1.77 n_0$

Higher T, lower is the probability to finish the cycle
Self-insurance

Beech stand

Beech stand

Growth : $G(s,n,t) = (1 - e^{-m_1 n\sqrt{4\pi s}})(m_2 + m_3 dH(t))$
where H is tree-high : $dH(t) = m_4 H_{100} e^{-m_4 t}$ (Dhôte 1995)

Volume Table :
$v(d,h) = (a_0 d^2 h + a_1 dh + a_2 d^3 h^2)(1 + \frac{b_1}{d^3} + b_2 d^2 + \frac{b_3}{h} + b_4 h)$
(Bouchon 1982)

Price function (Euro) : for $d > 7$ (in cm)
$p(s,t) = (12 + 45(1 - (1 - \frac{d}{65})_+^{1.8})v(d,h(t))$ (ajusted Tarp et al. 2000)

Beech stand

$\lambda = .01, \delta = .02, \overline{h} = .2$ in year^{-1}

Risk	$h(.)$	T	J_0
$\alpha = 0.6, \alpha_p = 0.4$, 4000 stems/ha, $H_{100} = 30$ m			
No	$h(t) = \overline{h}, t \in [0, 12.9] \cup [51.7, 57.3]$	81.9	$1.23 n_0$
Yes	$h(t) = \overline{h}, t \in [0, 13.1] \cup [44.0, 49.4]$	75.6 (53.0)	$0.67 n_0$
$\alpha = 0.6, \alpha_p = 0.4$, 4000 stems/ha, $H_{100} = 40$ m			
No	$h(t) = \overline{h}, t \in [0, 13.0] \cup [49.9, 55.5]$	75.3	$2.21 n_0$
Yes	$h(t) = \overline{h}, t \in [0, 13.0] \cup [42.4, 47.9]$	69.1 (49.9)	$1.47 n_0$

Respacing and thinning

Conclusion

- Harvesting increases with risk
- Larger harvesting can induces larger final term
- Final term can be larger with risk
- Self-insurance -> we make endogenous the risk through optimization
- Species dependency
- Influence of model complexity

THANK YOU FOR YOUR ATTENTION

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